ON A GRÜSS-LUPAS TYPE INEQUALITY AND ITS APPLICATION FOR THE ESTIMATION OF p-MOMENTS OF GUESSING MAPPINGS

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Abstract. An inequality of Grüss-Lupas type in normed spaces is proved. Some applications in estimating the p-moments of guessing mapping which complement the recent results of Massey [1], Arikan [2], Boztas [3] and Dragomir-van der Hoek [5]-[7] are also given.

1. Introduction

In 1935, G. Grüss proved the following integral inequality which gives an approximation of the integral of the product in terms of the product of integrals as follows

$$\begin{aligned}(1.1)\qquad & \left|\frac{1}{b-a}\int_a^b f(x)\,g(x)\,dx-\frac{1}{b-a}\int_a^b f(x)\,dx\cdot\frac{1}{b-a}\int_a^b g(x)\,dx\right| \\ \leq\ & \frac{1}{4}(\Phi-\varphi)(\Gamma-\gamma)\end{aligned}$$

where $f,g:[a,b]\to\mathbb{R}$ are integrable on $[a,b]$ and satisfying the assumption

$$\varphi\leq f(x)\leq\Phi,\gamma\leq g(x)\leq\Gamma \qquad (1.2)$$

for each $x\in[a,b]$ where $\varphi,\Phi,\gamma,\Gamma$ are given real constants.

Moreover, the constant $\frac{1}{4}$ is sharp in the sense that it can not be replaced by a smaller one.

For a simple proof of (1.1) as well as for some other integral inequalities of Grüss' type see the Chapter X of the recent book [4] by Mitrinović, Pečarić and Fink.

In 1950, M. Biernacki, H. Pidek and C. Ryll-Nardzewski established the following discrete version of Grüss' inequality [4, Chap. X]:

Theorem 1. *Let $a=(a_1,...,a_n)$, $b=(b_1,...,b_n)$ be two n-tuples of real numbers such that $r\leq a_i\leq R$ and $s\leq b_i\leq S$ for $i=1,...,n$. Then one has*

$$\begin{aligned}(1.3)\qquad & \left|\frac{1}{n}\sum_{i=1}^n a_ib_i-\frac{1}{n}\sum_{i=1}^n a_i\cdot\frac{1}{n}\sum_{i=1}^n b_i\right| \\ \leq\ & \frac{1}{n}\left[\frac{n}{2}\right]\left(1-\frac{1}{n}\left[\frac{n}{2}\right]\right)(R-r)(S-s)\end{aligned}$$

where $[x]$ is the integer part of $x,x\in\mathbb{R}$.

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A weighted version of Grüss' discrete inequality was proved by J.E. Pečarić in 1979, [4, Chap. X]:

Theorem 2. *Let a,b be two monotonic n-tuples and p a positive one. Then*

$$\left|\frac{1}{P_n}\sum_{i=1}^{n}p_ia_ib_i-\frac{1}{P_n}\sum_{i=1}^{n}p_ia_i\cdot\frac{1}{P_n}\sum_{i=1}^{n}p_ib_i\right| \leq |a_n-a_1|\,|b_n-b_1|\max_{1\leq k\leq n-1}\left(\frac{P_k\bar{P}_{k+1}}{P_n^2}\right) \tag{1.4}$$

where $P_n:=\sum_{i=1}^{n}p_i$, $\bar{P}_{k+1}=P_n-P_{k+1}$.

In 1981 , A. Lupas [4, Chap. X] proved some similar results for the first difference of a as follows :

Theorem 3. *Let a,b two monotonic n-tuples in the same sense and p a positive n-tuple. Then*

$$\begin{aligned}&\min_{1\leq i\leq n-1}|a_{i+1}-a_i|\min_{1\leq i\leq n-1}|b_{i+1}-b_i|\left[\frac{1}{P_n}\sum_{i=1}^{n}i^2p_i-\left(\frac{1}{P_n}\sum_{i=1}^{n}ip_i\right)^2\right] \\ \leq\ &\frac{1}{P_n}\sum_{i=1}^{n}p_ia_ib_i-\frac{1}{P_n}\sum_{i=1}^{n}p_ia_i\cdot\frac{1}{P_n}\sum_{i=1}^{n}p_ib_i \\ \leq\ &\max_{1\leq i\leq n-1}|a_{i+1}-a_i|\max_{1\leq i\leq n-1}|b_{i+1}-b_i|\left[\frac{1}{P_n}\sum_{i=1}^{n}i^2p_i-\left(\frac{1}{P_n}\sum_{i=1}^{n}ip_i\right)^2\right]\end{aligned} \tag{1.5}$$

If there exists the numbers $\bar{a},\bar{a}_1,r,r_1,(rr_1>0)$ such that $a_k=\bar{a}+kr$ and $b_k=\bar{a}_1+kr_1$, then in (1.5) the equality holds.

For some generalizations of Gruss' inequality for isotonic linear functionals defined on certain spaces of mappings see Chapter X of the book [4] where further references are given .

2. Some Grüss-Lupas Type Inequalities

The following inequality of Grüss-Lupas type in normed linear spaces holds:

Theorem 4. *Let $(X,\|\cdot\|)$ be a normed linear space over $K=(\mathbb{R},\mathbb{C})$, $x_i\in X$, $\alpha_i\in K$ and $p_i\geq 0$ $(i=1,...,n)$ such that $\sum_{i=1}^{n}p_i=1$. Then we have the inequality:*

$$\left\|\sum_{i=1}^{n}p_i\alpha_ix_i-\sum_{i=1}^{n}p_i\alpha_i\cdot\sum_{i=1}^{n}p_ix_i\right\| \leq \max_{1\leq j\leq n-1}|\alpha_{j+1}-\alpha_j|\max_{1\leq j\leq n-1}\|x_{j+1}-x_j\|\left[\sum_{i=1}^{n}i^2p_i-\left(\sum_{i=1}^{n}ip_i\right)^2\right]. \tag{2.1}$$

The inequality (2.1) is sharp in the sense that the constant $C=1$ in the right membership cannot be replaced by a smaller one.

Proof. Let us start with the following identity which can be proved by direct computation:

$$\begin{aligned} & \sum_{i=1}^{n} p_i \alpha_i x_i - \sum_{i=1}^{n} p_i \alpha_i \sum_{i=1}^{n} p_i x_i \\ = & \frac{1}{2} \sum_{i,j=1}^{n} p_i p_j \left(\alpha_j - \alpha_i\right)\left(x_j - x_i\right) \\ = & \sum_{1 \le i < j \le n}^{n} p_i p_j \left(\alpha_j - \alpha_i\right)\left(x_j - x_i\right). \end{aligned}$$

As $i < j$, we can write that

$$\alpha_j - \alpha_i = \sum_{k=i}^{j-1} \left(\alpha_{k+1} - \alpha_k\right)$$

and

$$x_j - x_i = \sum_{k=i}^{j-1} \left(x_{k+1} - x_k\right).$$

Using the generalized triangle inequality we have successively:

$$\begin{aligned} & \left\| \sum_{i=1}^{n} p_i \alpha_i x_i - \sum_{i=1}^{n} p_i \alpha_i \sum_{i=1}^{n} p_i x_i \right\| \\ = & \left\| \sum_{1 \le i < j \le n}^{n} p_i p_j \sum_{k=i}^{j-1} \left(\alpha_{k+1} - \alpha_k\right) \sum_{k=i}^{j-1} \left(x_{k+1} - x_k\right) \right\| \\ \le & \sum_{1 \le i < j \le n}^{n} p_i p_j \left| \sum_{k=i}^{j-1} \left(\alpha_{k+1} - \alpha_k\right) \right| \left\| \sum_{k=i}^{j-1} \left(x_{k+1} - x_k\right) \right\| \\ \le & \sum_{1 \le i < j \le n}^{n} p_i p_j \sum_{k=i}^{j-1} \left|\alpha_{k+1} - \alpha_k\right| \sum_{k=i}^{j-1} \left\|x_{k+1} - x_k\right\| =: A. \end{aligned}$$

Note that

$$\left|\alpha_{k+1} - \alpha_k\right| \le \max_{1 \le s \le n-1} \left|\alpha_{s+1} - \alpha_s\right|$$

and

$$\left\|x_{k+1} - x_k\right\| \le \max_{1 \le s \le n-1} \left\|x_{s+1} - x_s\right\|$$

for all $k = i, ..., j-1$ and then by summation,

$$\sum_{k=i}^{j-1} \left|\alpha_{k+1} - \alpha_k\right| \le (j-i) \max_{1 \le s \le n-1} \left|\alpha_{s+1} - \alpha_s\right|$$

and

$$\sum_{k=i}^{j-1} \left\|x_{k+1} - x_k\right\| \le (j-i) \max_{1 \le s \le n-1} \left\|x_{s+1} - x_s\right\|.$$

Taking into account the above estimations, we can write

$$A \leq \left[\sum_{1 \leq i < j \leq n}^{n} p_i p_j (j-i)^2 \right] \max_{1 \leq s \leq n-1} |\alpha_{s+1} - \alpha_s| \max_{1 \leq s \leq n-1} \|x_{s+1} - x_s\| .$$

As a simple calculation shows that

$$\sum_{1 \leq i < j \leq n}^{n} p_i p_j (j-i)^2 = \sum_{i=1}^{n} i^2 p_i - \left(\sum_{i=1}^{n} i p_i \right)^2$$

the inequality (2.1) is proved.
Assume that the inequality (2.1) holds with a constant $c > 0, i.e.,$

$$\begin{aligned} &\left\| \sum_{i=1}^{n} p_i \alpha_i x_i - \sum_{i=1}^{n} p_i \alpha_i \sum_{i=1}^{n} p_i x_i \right\| \\ \leq \ & c \max_{1 \leq j \leq n-1} |\alpha_{j+1} - \alpha_j| \max_{1 \leq j \leq n-1} \|x_{j+1} - x_j\| \left[\sum_{i=1}^{n} i^2 p_i - \left(\sum_{i=1}^{n} i p_i \right)^2 \right] . \end{aligned} \tag{2.2}$$

Now, choose the sequences $\alpha_k = \alpha + k\beta \, (\beta \neq 0)$, $x_k = x + ky \, (y \neq 0)$ $(k = 1, ..., n)$. We get

$$\begin{aligned} &\left\| \sum_{i=1}^{n} p_i \alpha_i x_i - \sum_{i=1}^{n} p_i \alpha_i \sum_{i=1}^{n} p_i x_i \right\| \\ = \ & \frac{1}{2} \left\| \sum_{i,j=1}^{n} p_i p_j (i-j)^2 \beta y \right\| = |\beta| \, \|y\| \left[\sum_{i=1}^{n} i^2 p_i - \left(\sum_{i=1}^{n} i p_i \right)^2 \right] \end{aligned}$$

and

$$\begin{aligned} &\max_{1 \leq j \leq n-1} |\alpha_{j+1} - \alpha_j| \max_{1 \leq j \leq n-1} \|x_{j+1} - x_j\| \left[\sum_{i=1}^{n} i^2 p_i - \left(\sum_{i=1}^{n} i p_i \right)^2 \right] \\ = \ & |\beta| \, \|y\| \left[\sum_{i=1}^{n} i^2 p_i - \left(\sum_{i=1}^{n} i p_i \right)^2 \right] \end{aligned}$$

and then by (2.2) we get $c \geq 1$, which proves the sharpness of the constant $c = 1$. ∎

The following corollary holds:

Corollary 1. *Under the above assumptions for $\alpha_i, x_i \, (i = 1, ..., n)$ we have the inequality:*

$$\begin{aligned} &\left\| \frac{1}{n} \sum_{i=1}^{n} \alpha_i x_i - \frac{1}{n} \sum_{i=1}^{n} \alpha_i \cdot \frac{1}{n} \sum_{i=1}^{n} x_i \right\| \\ \leq \ & \frac{n^2 - 1}{12} \max_{1 \leq j \leq n-1} |\alpha_{j+1} - \alpha_j| \max_{1 \leq j \leq n-1} \|x_{j+1} - x_j\| . \end{aligned} \tag{2.3}$$

The constant $\frac{1}{12}$ is sharp in the sense that it cannot be replaced by a smaller one.

The proof follows by the above theorem, putting $p_i = \frac{1}{n}$ and taking into account that:

$$\sum_{i=1}^{n} i^2 p_i - \left(\sum_{i=1}^{n} i p_i \right)^2 = \frac{n^2 - 1}{12}.$$

3. Applications for the Moments of Guessing Mappings

J.L. Massey in [1] considered the problem of guessing the value of realization of random variable X by asking questions of the form: "Is X equal to x ? " until the answer is "Yes" .

Let $G(X)$ denote the number of guesses required by a particular guessing strategy when $X = x$.

Massey observed that $E(G(x))$, the average number of guesses, is minimized by a guessing strategy that guesses the possible values of X in decreasing order of probability.

We begin by giving a formal and generalized statement of the above problem by following E. Arikan [2].

Let (X, Y) be a pair of random variable with X taking values in a finite set χ of size n, Y taking values in a countable set $\mathcal{Y}$. Call a function $G(X)$ of the random variable X a *guessing function* for X if $G : \chi \to \{1, ..., n\}$ is one-to-one. Call a function $G(X \mid Y)$ a *guessing function for* X *given* Y if for any fixed value $Y = y, G(X \mid y)$ is a guessing function for X . $G(X \mid y)$ will be thought of as the number of guessing required to determine X when the value of Y is given.

The following inequalities on the moments of $G(X)$ and $G(X|Y)$ were proved by E. Arikan in the recent paper [2].

Theorem 5. *For an arbitrary guessing function* $G(X)$ *and* $G(X \mid Y)$ *and any* $p > 0$, *we have:*

$$E(G(X)^p) \geq (1 + \ln n)^{-p} \left[\sum_{x \in \chi} P_X(x)^{\frac{1}{1+p}} \right]^{1+p} \tag{3.1}$$

and

$$E(G(X \mid Y)^p) \geq (1 + \ln n)^{-p} \sum_{y \in \mathcal{Y}} \left[\sum_{x \in \chi} P_{X,Y}(x, y)^{\frac{1}{1+p}} \right]^{1+p} \tag{3.2}$$

where $P_{X,Y}$ *and* P_X *are probability distributions of* (X, Y) *and* X, *respectively.*

Note that, for $p = 1$, we get the following estimations on the average number of guesses:

$$E(G(X)) \geq \frac{\left[\sum_{x \in \chi} P_X(x)^{\frac{1}{2}} \right]^2}{1 + \ln n}$$

and

$$E(G(X)) \geq \frac{\sum_{y \in \mathcal{Y}} \left[\sum_{x \in \chi} P_{X,Y}(x, y)^{\frac{1}{2}} \right]^2}{1 + \ln n}.$$

In paper [3], Boztas proved the following analytic inequality and applied it for the moments of guessing mappings:

Theorem 6. *The relation*

$$\left[\sum_{k=1}^{n} p_k{}^{\frac{1}{r}}\right]^{r} \geq \sum_{k=1}^{n} \left(k^r - (k-1)^r\right) p_k \tag{3.3}$$

where $r \geq 1$ holds for any positive integer n, provided that the weights $p_1, ..., p_n$ are nonnegative real numbers satisfying the condition:

$$p_{k+1}^{\frac{1}{r}} \leq \frac{1}{k}\left(p_1^{\frac{1}{r}} + ... + p_k^{\frac{1}{r}}\right), k = 1, 2, ...n-1 \tag{3.4}$$

To simplify the notation further, we assume that the x_i are numbered such that x_k is always the k^{th} guess. This yields:

$$E\left(G^p\right) = \sum_{k=1}^{n} k^p p_k, p \geq 0.$$

If we now consider the guessing problem, we note that (3.1) can be written as [3]:

$$\left[\sum_{k=1}^{n} p_k^{\frac{1}{1+p}}\right]^{1+p} \geq E\left(G^{1+p}\right) - E\left((G-1)^{1+p}\right)$$

for guessing sequences obeying (3.4).

In particular, using the binomial expansion of $(G-1)^{1+p}$ we have the following corollary [3] :

Corollary 2. *For guessing sequences obeying* (3.4) *with* $r = 1+m$ *, the* m^{th} *guessing moment, when* $m \geq 1$ *is an integer satisfies:*

$$\begin{aligned} & E\left(G^m\right) \\ \leq\ & \frac{1}{1+m}\left[\sum_{k=1}^{n} p_k^{\frac{1}{1+m}}\right]^{1+m} \\ & +\frac{1}{1+m}\left\{\begin{pmatrix} m+1 \\ 2 \end{pmatrix} E\left(G^{m-1}\right) - \begin{pmatrix} m+1 \\ 3 \end{pmatrix} E\left(G^{m-2}\right) + ... + (-1)^{m+1}\right\}. \end{aligned} \tag{3.5}$$

The following inequalities immediately follow from Corollary 2:

$$E(G) \leq \frac{1}{2}\left[\sum_{k=1}^{n} p_k^{\frac{1}{2}}\right]^2 + \frac{1}{2}$$

and

$$E\left(G^2\right) \leq \frac{1}{3}\left[\sum_{k=1}^{n} p_k^{\frac{1}{3}}\right]^3 + E(G) - \frac{1}{3}.$$

We are able now to point out some new results for the p-moment of guessing mapping as follows.

Using Pečarić's result (1.4), we can state the following inequality for the moments of a guessing mapping $G(X)$:

Theorem 7. *Let $p, q > 0$. then we have the inequality:*

$$\begin{aligned} 0 &\leq E\left(G^{p+q}\right) - E\left(G^{p}\right) E\left(G^{q}\right) \\ &\leq \left(n^{p}-1\right)\left(n^{q}-1\right) \max_{n=\overline{1,n-1}} \left\{P_k\left(1-P_k\right)\right\} \end{aligned} \tag{3.6}$$

where $P_k = \sum_{i=1}^{k} p_i$.

Proof. Define the sequences $a_i = i^p$, $b_i = i^q$ which are monotonous nondecreasing. Using both Čebyšev's and Pečarić's results we can state

$$\begin{aligned} 0 &\leq \sum_{i=1}^{n} i^{p+q} p_i - \sum_{i=1}^{n} i^{p} p_i \sum_{i=1}^{n} i^{q} p_i \\ &\leq \left(n^{p}-1\right)\left(n^{q}-1\right) \max_{n=\overline{1,n-1}} \left\{P_k\left(1-P_k\right)\right\} \end{aligned}$$

which is exactly (3.6). ∎

Now, let us define the mappings m_n, $M_n : (0,\infty) \longrightarrow (0,\infty)$ given by

$$m_n(t) := \begin{cases} n^t - (n-1)^t & \text{if } t \in (0,1) \\ 2^t - 1 & \text{if } t \in [1,\infty) \end{cases}$$

and

$$M_n(t) := \begin{cases} 2^t - 1 & \text{if } t \in (0,1) \\ n^t - (n-1)^t & \text{if } t \in [1,\infty) \end{cases} .$$

Now, using Lupas' result (see Theorem 3) we can state the following result

Theorem 8. *Let $p, q > 0$. Then we have the inequality*

$$\begin{aligned} & m_n(p)\, m_n(q) \left[E\left(G^2\right) - E^2(G)\right] \\ \leq\ & E\left(G^{p+q}\right) - E\left(G^{p}\right) E\left(G^{q}\right) \\ \leq\ & M_n(p)\, M_n(q) \left[E\left(G^2\right) - E^2(G)\right]. \end{aligned} \tag{3.7}$$

Proof. Consider the sequences $a_i = i^p$, $b_i = i^q$ in Lupas' theorem (note that a_i, b_i are monotonous nondecreasing) to get:

$$\begin{aligned} & \min_{1\leq i\leq n-1} \left[(i+1)^p - i^p\right] \min_{1\leq i\leq n-1} \left[(i+1)^q - i^q\right] \left[E\left(G^2\right) - E^2(G)\right] \\ \leq\ & E\left(G^{p+q}\right) - E\left(G^{p}\right) E\left(G^{q}\right) \\ \leq\ & \max_{1\leq i\leq n-1} \left[(i+1)^p - i^p\right] \min_{1\leq i\leq n-1} \left[(i+1)^q - i^q\right] \left[E\left(G^2\right) - E^2(G)\right]. \end{aligned} \tag{3.8}$$

Now, let us observe that if $p \in (0,1)$, then the sequence $\alpha_i = i^p$ is concave, i.e.,

$$\alpha_{i+1} - \alpha_i \leq \alpha_i - \alpha_{i-1} \text{ for all } i = 2, ..., n-1$$

and if $p \in [1,\infty)$ then $\alpha_i = i^p$ is convex, i.e.,

$$\alpha_{i+1} - \alpha_i \geq \alpha_i - \alpha_{i-1} \text{ for all } i = 2, ..., n-1.$$

Consequently

$$\min_{1\leq j\leq n-1} \left[(j+1)^p - j^p\right] = m_n(p)$$

and

$$\max_{1\leq j\leq n-1} \left[(j+1)^p - j^p\right] = M_n(p).$$

Using (3.8) we get the desired inequality (3.7). ∎

Now, for a given $p>0$, consider the sum

$$S_p(n) := \sum_{i=1}^{n} i^p.$$

We know that

$$S_1(n) = \frac{n(n+1)}{2},$$

$$S_2(n) = \frac{n(n+1)(2n+1)}{6}$$

and

$$S_3(n) = \left[\frac{n(n+1)}{2}\right]^2.$$

Using Biernaki-Pidek-Nardzewski's result (see Theorem 1) we can state and prove the following approximation result concerning the p -moment of guessing mapping $G(X)$.

Theorem 9. *Let $p>0$. Then we have the estimation*

$$\left|E\left(G^p(X)\right) - \frac{1}{n}S_p(n)\right| \le \left[\frac{n}{2}\right]\left(1-\frac{1}{n}\left[\frac{n}{2}\right]\right)(n^p-1)(p_M-p_m)$$

where $p_M = \max\{p_i \mid i=1,...,n\}$ and $p_m := \min\{p_i \mid i=1,...,n\}$.

Proof. Let us choose in Theorem 1, $a_i = p_i$, $b_i = i^p$. Then $p_m \le a_i \le p_M$, $1 \le b_i \le n^p$ for all $i=1,...,n$ and by (1.3) we get

$$\begin{aligned} & \left|\sum_{i=1}^{n} i^p p_i - \frac{1}{n}\sum_{i=1}^{n} i^p \sum_{i=1}^{n} p_i\right| \\ \le\ & \left[\frac{n}{2}\right]\left(1-\frac{1}{n}\left[\frac{n}{2}\right]\right)(n^p-1)(p_M-p_m) \end{aligned}$$

which proves the theorem. ∎

Remark 1. 1. *If in (3.5) we put $p=1$, we get*

$$\left|E(G(X)) - \frac{n+1}{2}\right| \le (n-1)\left[\frac{n}{2}\right]\left(1-\frac{1}{n}\left[\frac{n}{2}\right]\right)(p_M-p_m) \tag{3.9}$$

which is an estimation of the average number of guesses in term of the size n of X and $p_M - p_m$.

2. *Note that if $p=(p_1,...,p_n)$ is close to the uniform distribution $\left(\frac{1}{n},...,\frac{1}{n}\right)$, i.e.,*

$$0 \le p_M - p_m \le \frac{\varepsilon}{(n-1)\left[\frac{n}{2}\right]\left(1-\frac{1}{n}\left[\frac{n}{2}\right]\right)}, \varepsilon > 0 \tag{3.10}$$

then the error of approximating $E(G(X))$ by $\frac{n+1}{2}$ is less than $\varepsilon>0$.

Now, using our new inequality in Corollary 1 we shall be able to prove another type of estimation for the p-moment of guessing mapping $G(X)$ as follows:

Theorem 10. *Let $p > 0$. Then we have the estimation:*

$$\left| E\left(G^{p}\left(X\right)\right) - \frac{1}{n} S_{p}\left(n\right) \right| \leq \frac{\left(n^{2}-1\right) n}{12} M_{n}\left(p\right) \max_{j=\overline{1,n-1}} \left|p_{j+1} - p_{j}\right|. \tag{3.11}$$

Proof. Follows by Corollary 1, choosing $\alpha_i = i^p, x_i = p_i$ and $\|\cdot\|$ is the usual modulus $|\cdot|$ from the real number field $\mathbb{R}$. ∎

Remark 2. 1. *If in* (3.11) *we put $p = 1$, we get*

$$\left| E\left(G^{p}\left(X\right)\right) - \frac{n+1}{2} \right| \leq \frac{n\left(n^{2}-1\right)}{12} \max_{j=\overline{1,n-1}} \left|p_{j+1} - p_{j}\right|, \tag{3.12}$$

which is another type of estimation for the average number of guesses in terms of the size of X and of the "step size" of probabilities p_i.

2. *Note that if we choose*

$$\max_{j=\overline{1,n-1}} \left|p_{j+1} - p_{j}\right| < \frac{12\varepsilon}{n\left(n^{2}-1\right)}, \varepsilon > 0$$

then

$$\left| E\left(G^{p}\left(X\right)\right) - \frac{n+1}{2} \right| < \varepsilon.$$

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